Autonomous Navigation for Flying Robots

Lecture 4.3 : Kinematics and Dynamics

Jürgen Sturm
Technische Universität München

Kinematics

- Describes the motion of rigid bodies
- Position
- Velocity
- Acceleration

Example: 1D Kinematics

- State $\mathbf{x} = \begin{pmatrix} x & \dot{x} & \ddot{x} \end{pmatrix}^\top \in \mathbb{R}^3$
- Action $u \in \mathbb{R}$
- Time constant $\Delta t \in \mathbb{R}$
- Linear process model

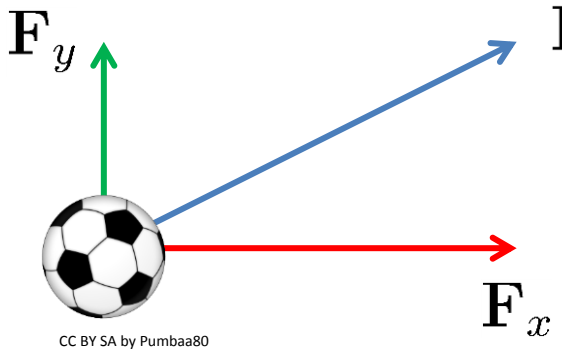
$$\mathbf{x}_t = \begin{pmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{pmatrix} \mathbf{x}_{t-1} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_t$$

Dynamics

- Actuators induce forces and torques
- Forces induce linear acceleration
- Torques induce angular acceleration

Forces and Accelerations

- Forces are vectors and can be summed up
- Important forces (for us): Gravity, thrust, friction
- Forces induce accelerations



$$m\ddot{\mathbf{x}} = \mathbf{F}$$

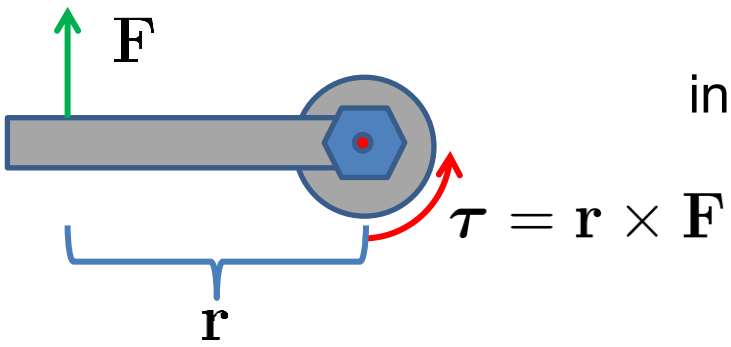
mass

Torques and Angular Accelerations

- Force on a lever induces a torque (“turning force”)
- Forces are vectors and can be summed up
- Torque results in angular acceleration $\boldsymbol{\alpha}$

$$J\boldsymbol{\alpha} = \boldsymbol{\tau}$$

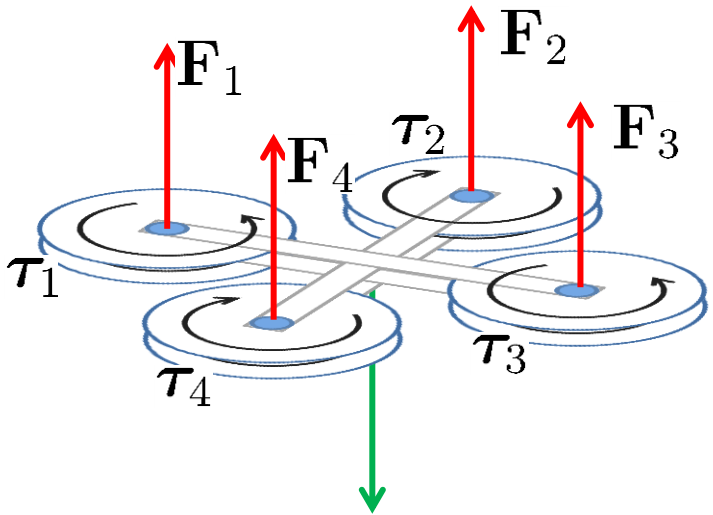
inertial matrix



$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

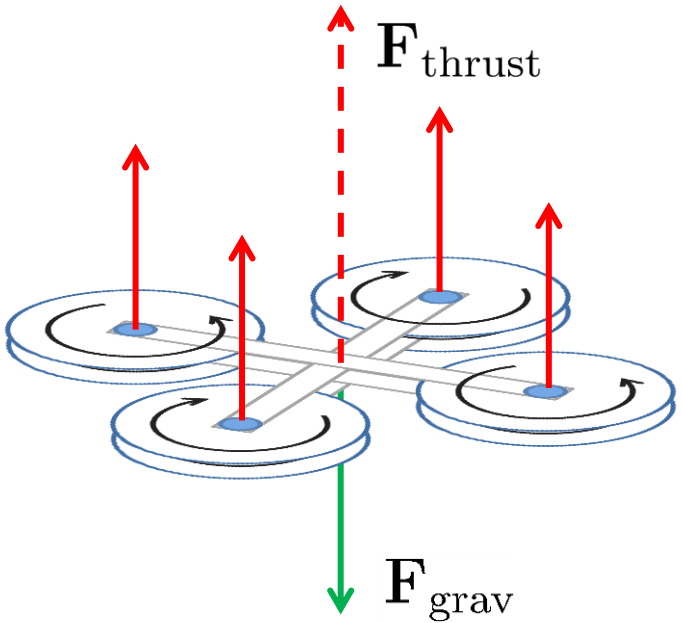
Dynamics of a Quadrotor

- Each propeller induces force and torque by accelerating air
- Gravity pulls quadrocopter downwards



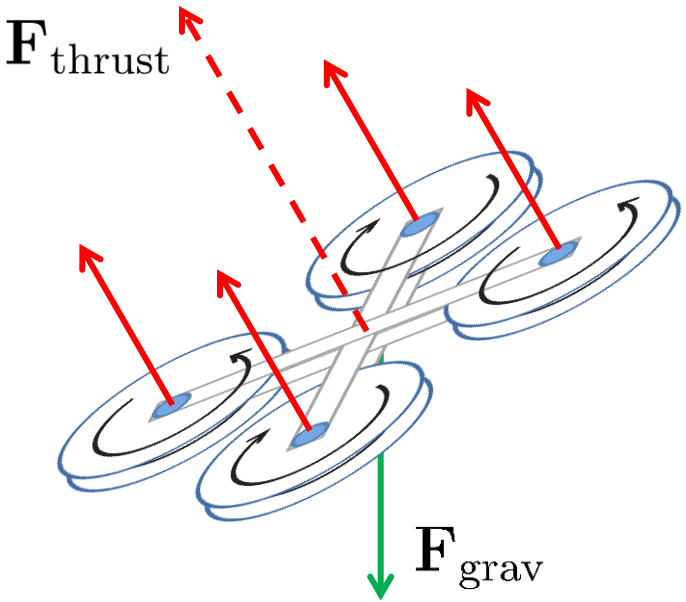
Vertical Acceleration

- Thrust $\mathbf{F}_{\text{thrust}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$



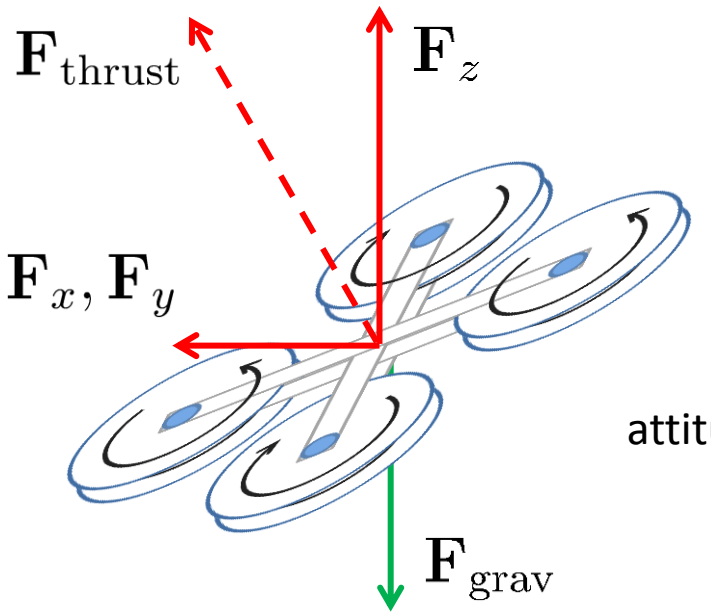
Vertical and Horizontal Acceleration

- Thrust $\mathbf{F}_{\text{thrust}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$



Vertical and Horizontal Acceleration

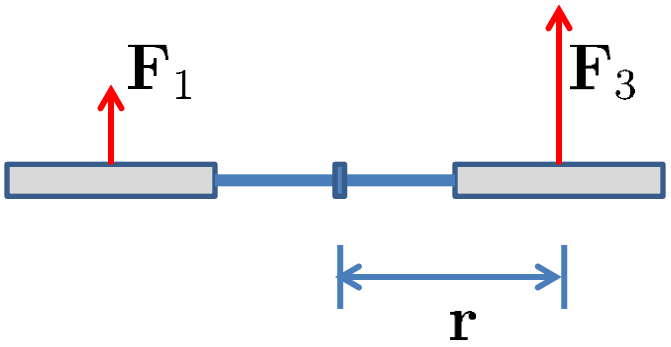
- Thrust $\mathbf{F}_{\text{thrust}} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$
- Acceleration $\ddot{\mathbf{x}}_{\text{global}} = (R_{RPY}\mathbf{F}_{\text{thrust}} - \mathbf{F}_{\text{grav}})/m$



attitude R_{RPY}

Pitch (and Roll)

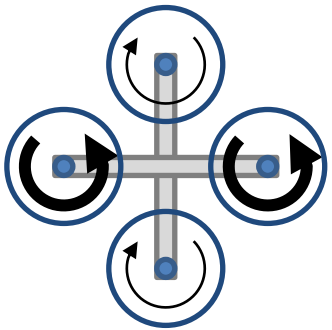
- Attitude changes when opposite motors generate unequal thrust
- Induced torque $\boldsymbol{\tau} = (F_1 - F_3) \times \mathbf{r}$
- Induced angular acceleration $\boldsymbol{\alpha} = J^{-1}\boldsymbol{\tau}$



Side view of quadrotor

Yaw

- Each propeller induces torque due to rotation and the interaction with the air
- Induced torque
- Induced angular acceleration $\boldsymbol{\tau} = \boldsymbol{\tau}_1 - \boldsymbol{\tau}_2 + \boldsymbol{\tau}_3 - \boldsymbol{\tau}_4$



Lessons Learned

- Rigid body kinematics:
  Position, velocity, acceleration
- Dynamics:
  Forces and torques
- Application to quadrotors